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International Baccalaureate®
Baccalauréat International
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Markscheme

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Mathematical studies

Standard level

Paper 1

24 pages

International Baccalaureate®
Baccalauréat International
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Paper 1 Markscheme
Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6.

1   Abbreviations

The markscheme may make use of the following abbreviations:

M   Marks awarded for **Method**

A   Marks awarded for an **Answer** or for **Accuracy**

C   Marks awarded for **Correct** answers (irrespective of working shown)

R   Marks awarded for clear **Reasoning**

ft   Marks that can be awarded as **follow through** from previous results in the question

2   Method of Marking

(a)   All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b)   If the candidate has full marks on a question use the **C6** annotation, if the candidate has made an attempt but scores zero marks use **C0**. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.

(c)   In this paper, if the **correct answer is seen on the answer line the** maximum mark is awarded. **There is no need to check the working!** Award **C** marks and move on.

(d)   If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.

(e)   If the **answer is wrong**, marks should be awarded for the working according to the markscheme.

(f)   Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.

(g)   A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.

(h)   If correct working results in a correct answer **in the working box** but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final *(A1)* *(ignore the further working)*
2.	$(x-6)(x+1)$	$x = 6$ and -1	Do **not** award the final *(A1)* *(see next example)*

Example: Factorise $x^2 - 5x - 6$

Markscheme	Candidates' Scripts		Marking
$(x-6)(x+1)$ **(A1)(A1)**	(i)	Answer line: $(x+6)(x+1)$	**(A0)(A1)**
	(ii)	Working box: $(x-6)(x+1)$	**(A1)**
		followed by $x=6$ and -1, or just $6, -1$	
		in either working box or on answer line.	**(A0)**

3   Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded.   Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a)   Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b)   If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.

(c)   If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d)   To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e)   The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f)   Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ **(M1)(A1)**	(a) $\dfrac{\sin A}{4} = \dfrac{\sin 30}{3}$	**(M1)(A0)**
		(use of sine rule but with wrong values)
$A = 22.0°\ (22.0243\ldots)$ **(A1)**	$A = 41.8°$	**(A0)**
		*(**Note:** the 2nd **(A1)** here was not marked **(ft)** and cannot be awarded because there was an earlier error in the **same** question part.)*
(b) $x = 7\tan(22.0243\ldots°)$ **(M1)**	(b) case (i) $x = 7\tan 41.8°$	**(M1)**
$= 2.83\ (2.83163\ldots)$ **(A1)(ft)**	$= 6.26$	**(A1)(ft)**
	but case (ii) 6.26	**(C0)**
		since no working shown

4   Using the Markscheme

(a)   **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**.   Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.
        The only exception will be for an answer where the accuracy is specified in the question – see section 5.

(b)   **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**.   Hence the **(A1)** is not awarded for a correct answer if no reason or the wrong reason is given.

(c)   **Alternative methods** may not always be included.   Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
        Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(d)   Unless the question specifies otherwise, accept **equivalent forms**.   For example: $\dfrac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

        On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
        Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
             the 3 significant figure answer worked through from full calculator display;

             the exact value $\left(\text{for example } \dfrac{2}{3} \text{ if applicable}\right)$;

             the full calculator display in the form $2.83163\ldots$ as in the example above.
        Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(e)   As this is an international examination, all valid **alternative forms of notation** should be accepted.   Some examples of these are:

        Decimal points: 1.7; $1'7$; $1\cdot 7$; $1,7$.

        Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or $.49$.

        Different descriptions of an interval: $3 < x < 5$; $(3, 5)$; $]\,3, 5\,[$.

        Different forms of notation for set properties (*e.g.* complement): A'; $\overline{A}$; A^c; $U - A$; $(A\,; U \setminus A.$

        Different forms of logic notation: $\neg p$; p'; $\tilde{p}$; $\overline{p}$; $\sim p$; $-p$.
                                              $p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

        Significance level may be written as α .

(f)   Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5   Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award *(A1)* and ignore subsequent rounding.
Note: The unrounded answer may appear in either the working box or on the final answer line.

2. If the candidate's unrounded answer is **not** seen then award *(A1)* if the answer given is **correctly** rounded to 2 or more significant figures, otherwise *(A0)*.
Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final *(A1)* irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	*(A1)*	*(A1)*	*(A0)*	*(A1)*	*(A0)*	*(A0)*
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

Examples:

Markscheme	Candidates' Scripts		Marking
9.43 (9.43398…) **(A1)**	(i)	9.43398… is seen in the working box followed by 9; 9.4; 9.43; 9.434 *etc.* (correctly rounded)	**(A1)**
	(ii)	9.43398… is seen in the working box followed by 9.433; 9.44 *etc.* (incorrectly rounded)	**(A1)**
	(iii)	9.4	**(A1)**
	(iv)	9	**(A0)** *(correct to 1 sf)*
	(v)	9.3	**(A0)** *(incorrectly rounded to 2 sf)*
	(vi)	9.44	**(A0)** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts		Marking
7.44 (7.43798…) **(A1)**	(i)	7.43798… is seen in the working box followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	**(A1)**
	(ii)	7.43798… is seen in the working box followed by 7.437; 7.43 *etc.* (incorrectly rounded)	**(A1)**
	(iii)	7.4	**(A1)**
	(iv)	7	**(A0)** *(correct to 1 sf)*
	(v)	7.5	**(A0)** *(incorrectly rounded to 2 sf)*
	(vi)	7.43	**(A0)** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $ABC = 90°$, $AC = 32\,cm$ and $AB = 30\,cm$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ **(M1)** *Award **(M1)** for correct substitution in Pythagoras' formula* $= 11.1\left(\sqrt{124}, 11.1355...\right)(cm)$ **(A1)**	(a) $BC = \sqrt{32^2 - 30^2}$ $11\,(cm)$ *(2 sf answer only seen, but correct)*	**(M1)** **(A1)**
(b) $Area = \dfrac{1}{2} \times 30 \times 11.1355...$ **(M1)** *Award **(M1)** for correct substitution in area of triangle formula* $= 167(167.032...)(cm^2)$ **(A1)(ft)**	(b) case (i) $Area = \dfrac{1}{2} \times 30 \times 11$ $= 165\,(cm^2)$ case (ii) $= 165\,(cm^2)$ *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*	**(M1)** *(working shown)* **(A1)(ft)** **(M0)(A0)(ft)**

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\dfrac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts		Marking
Chi-squared	(a)	7.7	**(A2)**
$7.68\,(7.67543\ldots)$ **(A2)**	(b)	7.67	**(A1)**
	(c)	7.6	**(A1)**
	(d)	8	**(A0)**
	(e)	7	**(A0)**
	(e)	7.66	**(A0)**

Regression line

Markscheme	Candidates' Scripts		Marking
$y = 0.888x + 13.5$ **(A2)** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most **(A1)(A0)**.	(a)	$y = 0.89x + 13$	**(A2)** (both accepted)
	(b)	$y = 0.88x + 13$	**(A1)** (one rounding error)
	(c)	$y = 0.88x + 14$	**(A1)** (rounding error repeated)
	(d) (i)	$y = 0.9x + 13$	
	(ii)	$y = 0.8x + 13$	**(A1)** (1 sf not accepted)
	(e)	$0.88x + 14$	**(A0)** (two rounding errors and not an equation)

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts		Marking
$(2.06, 4.49)$ **(A1)(A1)** $(2.06020\ldots, 4.49253\ldots)$	(a)	$(2.1, 4.5)$	**(A1)(A1)** (both accepted)
	(b)	$(2.0, 4.4)$	**(A1)** (same rounding error twice)
	(c)	$(2.06, 4.4)$	**(A1)** (one rounding error)
	(d)	$(2, 4.4)$	**(A0)** (1sf not accepted, one rounding error)

6 **Level of accuracy in finance questions**

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts		Marking
$231.62 (231.6189) **(A1)**	(i)	231.6	**(A0)**
	(ii)	232	**(A0)** *(Correct rounding to incorrect level)*
	(iii)	231.61	**(A0)**
	(iv)	232.00	**(A0)** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 **Units in answers**

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

Markscheme			Candidates' Scripts		Marking
(a)	$37000 \, \text{m}^2$	**(A1)**	(a)	$36000 \, \text{m}^2$	**(A0)** *(Incorrect answer so units not considered)*
(b)	$3200 \, \text{m}^3$	**(A1)**	(b)	$3200 \, \text{m}^2$	**(A0)** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

8 **Graphic Display Calculators**

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

1.

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
$-\dfrac{3}{11}$			✓	✓
42	✓	✓	✓	✓
3.14			✓	✓
$\sqrt{23}$				✓
-113		✓	✓	✓
$\dfrac{24}{6}$	✓	✓	✓	✓
$-5\dfrac{1}{5}$			✓	✓

Note: Award *(A1)* for each completely correct row, *(A0)* otherwise.

Total [6 marks]

2. (a) 40 (euros)   *(A1)*   *(C1)*
                                                                                [1 mark]

(b) $\left(M(94)=\right)\ 5(94)+40$ *(M1)*

Note: Award *(M1)* for correct substitution of 94 into given function.

   510 (euros)   *(A1)*   *(C2)*
                                                                                [2 marks]

(c) 7 (T-shirts)   *(A1)*   *(C1)*
                                                                                [1 mark]

(d) $9p=5p+40$ *(M1)*

Note: Award *(M1)* for equating the given functions. Accept a sketch showing both functions.

   $(p=)\ 10$ (T-shirts)   *(A1)*   *(C2)*
                                                                                [2 marks]

Total [6 marks]

3.   (a)   **If** it is not a doggo **then** it is not a woofer   *(A1)(A1)*   *(C2)*

> **Note:** Award *(A1)* for If… then. Award *(A1)* for the two correct propositions in correct order.

                                                                                   [2 marks]

(b) $(\neg q \wedge r) \Rightarrow p$ *(A1)*   *(C1)*

> **Note:** Award *(A0)* for $\neg q \wedge r \Rightarrow p$.

                                                                                   [1 mark]

(c)

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$((p \Rightarrow q) \wedge (q \Rightarrow r)) \underline{\vee} (p \Rightarrow r)$
T	T	T	T	T	**T**	T	**F**
T	T	F	T	F	**F**	F	**F**
T	F	T	F	T	**F**	T	**T**
T	F	F	F	T	**F**	F	**F**
F	T	T	T	T	**T**	T	**F**
F	T	F	T	F	**F**	T	**T**
F	F	T	T	T	**T**	T	**F**
F	F	F	T	T	**T**	T	**F**

                                                             (A1)(A1)(ft)   *(C2)*

> **Note:** Award *(A1)* for each correct column. Last column follows through from the previous column.

                                                                                   [2 marks]

(d)   not all / only some of the entries in the **final (or last)** column are false   *(R1)*   *(C1)*

       OR

       some of the entries in the **final (or last)** column are true   *(R1)*   *(C1)*

> **Note:** The final (or last) column must be explicitly referenced for the *(R1)* to be awarded.

                                                                                   [1 mark]

                                                                           Total [6 marks]

4.   (a)   4   *(A1)*   *(C1)*
                                                                                 [1 mark]

     (b)
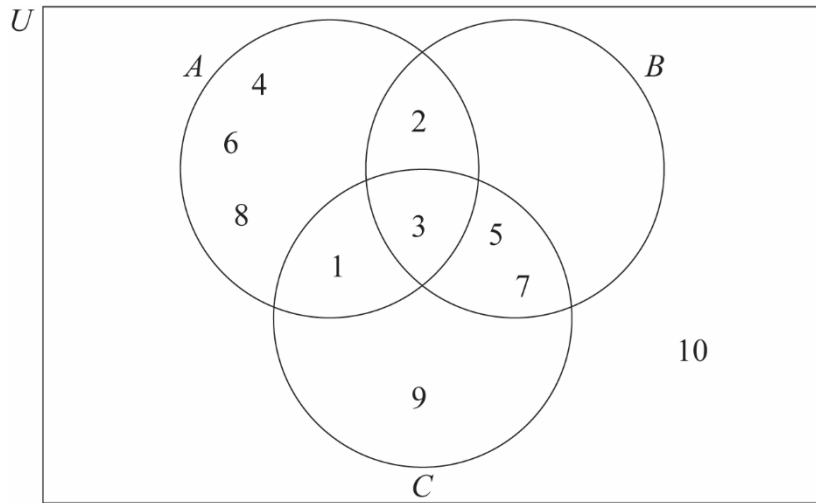
                                                                  (A1)(A1)(A1)(A1)   *(C4)*

Note:	Award *(A1)* for 3 in the correct place. Award *(A1)* for 1, 2, 5 and 7 in the correct places. Award *(A1)* for 4, 6, 8, 9 in the correct places. Award *(A1)* for 10 outside of the three circles **and** 11 not shown in the diagram.
	If any entry is duplicated within its region, award at most *(A3)*.

                                                                                 [4 marks]

     (c)   9   *(A1)(ft)*   *(C1)*

Note:	Award *(A1)* for the correct element. Follow through from *their* Venn diagram in part (b). Award *(A0)* if additional incorrect elements are included in their answer.

                                                                                 [1 mark]

                                                                           Total [6 marks]

5.   **units are required in both parts**

(a) $(V =) \; 5 \times 10 \times 40 + 2 \times 10 \times 40$ **(M1)(M1)**

> **Note:** Award **(M1)** for correct substitutions in volume formula for both cuboids.
>   Award **(M1)** for adding the volumes of both cuboids.

2800 cm^3 **(A1)   (C3)**

[3 marks]

(b) $2800 = \pi \times r^2 \times 30$ **(M1)(M1)**

> **Note:** Award **(M1)** for correct substitution in volume of cylinder formula. Award **(M1)**
>   for equating *their* expression (must include π and r) to *their* 2800.

$(r =) \; 5.45 \text{ cm} \; (5.45058\ldots \text{ cm})$ **(A1)(ft)   (C3)**

> **Note:** Follow through from *their* part (a).

[3 marks]

Total [6 marks]

6. (a) $y = 0.384x + 629$
$y = (0.384221\ldots)x + (629.421\ldots)$ **(A1)(A1)**   **(C2)**

> **Note:** Award **(A1)** for $0.384x$, **(A1)** for 629. If the answer is not given as an equation, award a maximum of **(A1)(A0)**.

[2 marks]

(b) $y = 0.384221\ldots \times 2900 + 629.421\ldots$ **(M1)**

> **Note:** Award **(M1)** for substitution into *their* regression equation.

1740 $(1744, 1743.66\ldots)$ (HKD) **(A1)(ft)**   **(C2)**

> **Note:** Follow through from part (a).

[2 marks]

(c) the correlation is (very) strong **(R1)**

2900 (km) is within the given data range (interpolation) **(R1)**   **(C2)**

> **Note:** Two correct reasons are required for the awarding of **(C2)**.

[2 marks]

Total [6 marks]

7.   (a) $9 = \left(\dfrac{8}{3}\right)r^3$ *(M1)*

> **Note:** Award *(M1)* for correctly substituted geometric sequence formula equated to 9.

   $(r =)\ 1.5\ \left(\dfrac{3}{2}\right)$ *(A1)*   *(C2)*

[2 marks]

(b)   4   *(A1)(ft)*   *(C1)*

> **Note:** Follow through from part (a).

[1 mark]

(c) $2500 < \dfrac{\left(\dfrac{8}{3}\right)\left((1.5)^k - 1\right)}{1.5 - 1}$ *(M1)*

> **Note:** Award *(M1)* for *their* correctly substituted geometric series formula compared to 2500.

   $k = 15.2\ (15.17319\ldots)$ *(A1)(ft)*

   $(k =)\ 16$ *(A1)(ft)*   *(C3)*

> **Note:** Answer must be an integer for the final *(A1)(ft)* to be awarded. Follow through from part (a).

[3 marks]

Total [6 marks]

8. (a) $FV = 50\,000\left(1 + \dfrac{5.6}{100 \times 12}\right)^{12 \times 4}$ **(M1)(A1)**

> **Note:** Award **(M1)** for substitution into compound interest formula, **(A1)** for correct substitution.

OR

$N = 4$
$I\% = 5.6$
$PV = (\mp)\,50\,000$
$P/Y = 1$
$C/Y = 12$ **(A1)(M1)**

> **Note:** Award **(A1)** for $C/Y = 12$ seen, **(M1)** for all other correct entries.

OR

$N = 48$
$I\% = 5.6$
$PV = (\mp)\,50\,000$
$P/Y = 12$
$C/Y = 12$ **(A1)(M1)**

> **Note:** Award **(A1)** for $C/Y = 12$ seen, **(M1)** for all other correct entries.

$(FV =)\,62\,520.97\ (\text{AUD})$ **(A1)** **(C3)**

> **Note:** The final **(A1)** is not awarded if the answer is not correct to 2 decimal places.

[3 marks]

Question 8 continued

(b) $\quad 2500 = 40\,000\left(1 - \dfrac{18}{100}\right)^{t}$ **OR** $\quad 2500 = 40\,000\,(0.82)^{t-1}$ **(M1)(A1)**

> **Note:** Award **(M1)** for substitution into compound interest formula or term of a geometric
> sequence formula and equating to 2500, **(A1)** for correct substitution.

OR

$I\% = -18$
$PV = (\pm)\,40\,000$
$FV = (\mp)\,2500$
$P/Y = 1$
$C/Y = 1$ **(A1)(M1)**

> **Note:** Award **(A1)** for $FV = (\mp)\,2500$ seen, **(M1)** for all other correct
> entries. PV and FV must have opposite signs.

OR

$32800, 26896, 22054.72, 18084.87, 14829.59, \ldots$ **(M1)**

$t_{13} = 3031.38$ and $t_{14} = 2485.73$ **(A1)**

> **Note:** Award **(M1)** for a list of at least 5 correct terms beginning with 32800,
> **(A1)** for identifying t_{13} and t_{14}.

$(t =)\ 14.0\ (13.9711\ldots)$ **(A1)** **(C3)**

[3 marks]

Total [6 marks]

9.   (a) $\dfrac{18}{24}\left(\dfrac{3}{4},\ 0.75,\ 75\%\right)$ *(A1)(A1)*   *(C2)*

> **Note:** Award *(A1)* for correct numerator, *(A1)* for correct denominator.

[2 marks]

(b) $\dfrac{18}{24}\times\dfrac{18}{24}$ *(M1)*

> **Note:** Award *(M1)* for the square of *their* probability in part (a).

$=0.563\left(\dfrac{9}{16},\ \dfrac{324}{576},\ 0.5625,\ 56.3\%\right)$ *(A1)(ft)*   *(C2)*

> **Note:** Follow through from part (a), provided *their* answer is less than or equal to 1.

[2 marks]

(c) $11\times\dfrac{18}{24}$ *(M1)*

> **Note:** Award *(M1)* for multiplying *their* part (a) by 11.

$8.25\left(\dfrac{33}{4}\right)$ *(A1)(ft)*   *(C2)*

> **Note:** Follow through from part (a), provided *their* answer is less than or equal to 1.

[2 marks]

Total [6 marks]

10.   (a) 460×3.07 ***(A1)(M1)***

> **Note:** Award ***(A1)*** for selecting 3.07 as the exchange rate, ***(M1)*** for multiplying 460 by an exchange rate from the table.

   1412 (MYR)   ***(A1)***   ***(C3)***

> **Note:** Do not award the final ***(A1)*** if the answer is to the wrong level of accuracy.

[3 marks]

   (b) $\dfrac{1412 - 440}{4.45}$ ***(M1)(M1)***

> **Note:** Award ***(M1)*** for *their* correct subtraction or for 972 (972.2) or *their* correct difference seen. Award ***(M1)*** for dividing by 4.45. Follow through from part (a).

   218 (USD)   ***(A1)(ft)***   ***(C3)***

> **Note:** Do not award the final ***(A1)*** if the answer is to the wrong level of accuracy.

[3 marks]

Total [6 marks]

11. (a) 170 *(A1)*   *(C1)*

[1 mark]

(b) $130\,000 = 170 \times 1.31^{t}$ *(M1)*

> **Note:** Award *(M1)* for equating $130\,000$ to the exponential function.

$(t =)$ 24.6 (24.5882… (years)) *(A1)*   *(C2)*

[2 marks]

(c) 170×1.31^{70} *(M1)*

> **Note:** Award *(M1)* for correct substitution in the function $N(t)$.

2.75×10^{10} ($2.75067… \times 10^{10}$, $27\,500\,000\,000$, $27\,506\,771\,343$) *(A1)*   *(C2)*

[2 marks]

(d) The number of Elvis impersonators in 2047,
is greater than the world population. *(R1)*   *(C1)*

  OR

$2.75 \times 10^{10} > 9\,500\,000\,000$ *(R1)*   *(C1)*

> **Note:** Award *(R1)* for a correct comparison of *their* number of impersonators with the world population. Follow through from part (c) if a reasonable argument can be made that the model is unrealistic.
>
> Award *(R0)* if the number of impersonators is not explicitly seen in part (c) or in part (d).

[1 mark]

Total [6 marks]

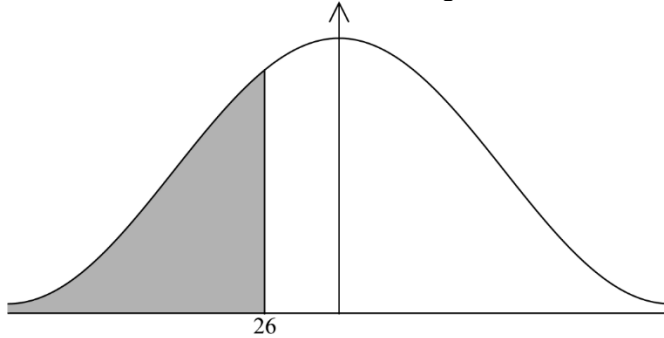**12.** (a) $0.5 \left(\dfrac{1}{2}, 50\% \right)$ *(A1)*   *(C1)*

[1 mark]

(b) $\mathrm{P}(X \leq 26)$ *(M1)*

> **Note:** Award *(M1)* for a correct mathematical statement.
> **OR**
> Award *(M1)* for a diagram that shows the value 26 labelled to the left of the mean and the correct shaded region.

       0.345 $(0.344578\ldots, 34.5\%)$ *(A1)*   *(C2)*

[2 marks]

(c) 0.7 **OR** 0.3 (seen)   *(A1)*

> **Note:** Award *(A1)* for 0.7 or 0.3 seen.

       $\mathrm{P}(\text{time} < k) = 0.7$ **OR** $\mathrm{P}(\text{time} > k) = 0.3$ *(M1)*

> **Note:** Award *(M1)* for a correct mathematical statement.
> **OR**
> Award *(M1)* for a diagram that shows k greater than the mean and shading in the region below k, above k, or between k and the mean.
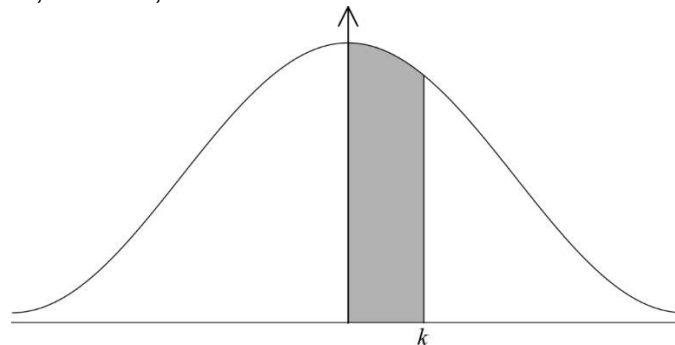
       $(k =)$ 30.6 $(30.6220\ldots)$ (minutes)   *(A1)*   *(C3)*

> **Note:** Accept "30 minutes and 37 seconds" or (from 3 sf k value) "30 minutes and 36 seconds".

[3 marks]

Total [6 marks]

13. (a)   continuous   *(A1)*   *(C1)*

[1 mark]

(b)   6   *(A1)*   *(C1)*

> **Note:** Award *(A0)* for an answer of $60 \leq w < 70$.

[1 mark]

(c)   8.97 (8.97479…) (g)   *(A2)*   *(C2)*

[2 marks]

(d)   [55.9, 73.9] **OR** $55.9252… \leq w \leq 73.8747…$ *(M1)*

> **Note:** Award *(M1)* for correct endpoints seen. If the answer to part (c) is 14.1421…,
> award *(M1)* for endpoints of [50.7578…, 79.0421…].

45   *(A1)(ft)*   *(C2)*

> **Note:** Follow through from *their* part (c). For a standard deviation between 0 and 5 inclusive,
> the *FT* answer is 0.

[2 marks]

Total [6 marks]

14. (a) $(k =) \, 0$ **(A1)**   **(C1)**

> **Note:**   Award **(A1)** for an answer of "$x = 0$".

[1 mark]

(b) $x - \dfrac{2a}{x^2}$ **(A1)(A1)(A1)**   **(C3)**

> **Note:**   Award **(A1)** for x, **(A1)** for $-2a$, **(A1)** for x^{-2} or $\dfrac{1}{x^2}$. Award at most **(A1)(A1)(A0)** if extra terms are seen.

[3 marks]

(c) $0.5 = 2 - \dfrac{2a}{2^2}$ **(M1)**

> **Note:**   Award **(M1)** for *their* correctly substituted derivative equated to 0.5.

     $(a =) \, 3$ **(A1)(ft)**   **(C2)**

> **Note:**   Follow through from part (b) providing their answer is **not** $a = 0$ as this value contradicts the graph.

[2 marks]

Total [6 marks]

15. (a)   42 (minutes)   **(A1)**   **(C1)**

[1 mark]

(b)   Stephen is correct.   **(A1)**

     the minimum rehearsal time is greater than zero   **(R1)**

     OR

     he rehearsed at least 20 minutes every day   **(R1)**   **(C2)**

> **Note:**   Do not award **(A1)(R0).** Accept equivalent reasoning based on the box-and-whisker diagram.

[2 marks]

(c) $0 \le k \le 15, \ k \in \mathbb{N}$ **OR** $0, 1, 2, 3, \ldots, 15$ **(A1)(A1)(A1)**   **(C3)**

> **Note:**   Award **(A1)(A1)** for each correct endpoint of the interval, **(A1)** for indication of whole values between *their* endpoints. Accept $k \in \mathbb{Z}$.

[3 marks]

Total [6 marks]